HOMEWORK # 3

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1. Homework problems

- 1. Let $Y = \{x_1^2 + y_1^2 + x_2^2 + y_2^2 = 1\} \subset \mathbb{R}^4$, Show that
$$\frac{1}{2}(x_1 dy_1 - y_1 dx_1 + x_2 dy_2 - y_2 dx_2)$$
restricts to a contact form on Y.
- 2. Let $Y = S^3$. Show that for any embedded Reeb orbit γ,
$$CZ(\gamma^d) \geq dCZ(\gamma) - d + 1,$$
where γ^d denotes the Reeb orbit obtained by traversing γ d-fold times. Using this, show that if Y is the boundary of a convex domain, and C is a once-punctured genus 0, index two pseudoholomorphic curve with a positive puncture at γ^d, then we must have $d = 1$.
- 3. Let $Y = S^3$ and let C be an index zero branched cover of a trivial cylinder T over a hyperbolic Reeb orbit. Show that C has no branch points and is itself a cylinder. (The assumption that $Y = S^3$ here is not necessary, but you can assume it for simplicity.) Hint: use the Riemann-Hurwitz formula, which says in this case that
$$\chi(C) = d\chi(T) - b,$$
where b is the number of branch points and d is the degree of the covering.
- 4. (Bonus) Describe the Reeb orbits for the Reeb vector field in Question 1.

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