

Final project suggestions

Below are some suggestions for your final projects. You are **not** required to pick a project from this list — you can come up with your own project, and this list is only meant to help guide you.

- (1) Explore some of the topics in Euclid's elements that we did not discuss in class. For example, Books 7 – 9 cover elementary number theory — explain some of what is said here.
- (2) Write about the five regular Platonic solids. How are they constructed? Why are they called Platonic? Have a look at Book 13 of Euclid.
- (3) Write more about which regular polygons are constructible. We discussed this in class, but go deeper.
- (4) Write more about some famous operations that can not be done by straight edge and compass. For example, what does it mean that we can not square the circle? Can you explain some of the ideas for why it is not possible? Or, what does it mean to not be able to trisect an angle?
- (5) Read the article “Elementary surprises in projective geometry”, by Schwartz and Tabachnikov. Summarize some of it.
- (6) How do you construct a regular 17-gon? There are resources online that can help you with this.
- (7) Write about some of the connections between projective geometry and computer vision. A nice article about this is “An introduction to Projective Geometry (for computer vision)” by Stan Birchfield.
- (8) Explore the higher dimensional projective spaces $\mathbb{R}P^n$. Can you say some interesting things about $\mathbb{R}P^3$, for example?

- (9) Explore the projective spaces over different fields, like $\mathbb{C}P^n$, or FP^n , where F is a more exotic field, for example a finite field. Can you report on some interesting facts?
- (10) In class, I explained a bit about how we can “visualize” $\mathbb{R}P^2$. Explain how to make this more precise. What does the video that I showed in class represent? Can you think about some other ways to visualize $\mathbb{R}P^2$? What about $\mathbb{R}P^3$? $\mathbb{C}P^2$?
- (11) One fact about the projective plane $\mathbb{R}P^2$ is that it is not “orientable”; the Möbius band is not orientable either. Explain what orientable means in this context, and try to explain why $\mathbb{R}P^2$ is not orientable.
- (12) The projective spaces $\mathbb{R}P^n$ are examples of manifolds. Explore this topic a bit. What is a manifold?
- (13) What are Grassmannians? How do they generalize projective space? Can you give some examples where some kind of visualization is possible?
- (14) A natural setting for geometry in modern mathematics is called Riemannian geometry. What is Riemannian geometry? Can you explain what $\mathbb{R}P^2$ is in this language? And/or give other illustrative examples?
- (15) Learn a bit about the geometry of surfaces, and write about it. One possible reference is another book by Stillwell, called “The geometry of surfaces.” There are numerous other resources online.
- (16) What are geodesics? How do they generalize what we have been discussing in class? Give some examples.
- (17) Explore ways to visualize four and five dimensional space. Can you explain some interesting shapes that exist in these spaces? For example, look up the “Klein bottle”.
- (18) Explore connections between hyperbolic geometry and art. For example, check out the four patterns by Escher “Circle Limit” *I – IV*. What do these have to do with hyperbolic geometry?
- (19) Learn about tessellations. What kind of tessellations of the Euclidean plane are possible? More interestingly, what about the hyperbolic plane? Draw some pictures!

- (20) Read the great article “The hyperbolic chamber” by Jon Leys and report back on it. It is about how one might create a museum to convey the experience of living in a hyperbolic world.
- (21) Write about the relationship between Möbius transformations of the plane, and transformations of the sphere, through the lens of stereographic projection. You might have a look at the beautiful youtube video “Möbius transformations revealed”.
- (22) Learn about the Farey-Ford tessellation, circle packing, and related topics; explain what you learned. Francis Bonahon has a lot of great material about this.
- (23) Make some computer applets for visualizing some of the concepts we have studied in class.
- (24) What is spherical geometry, and how is it related to projective geometry?
- (25) Explain more about the connection between projective geometry and perspective. Give a cogent account.
- (26) What is a “configuration space”? Give some examples for some interesting systems, and explain some of the geometry.