

Dear students,

I wanted to highlight some key points of the material on projective geometry, to help you study for the final.

Here are the key points I would like you to take away from what we discussed, with some sample problems. Our general reference for this is Chapter 5 of the book.

- First, make sure you understand the following key point: we are trying to find a geometry such that any two lines intersect at a unique point. Sample question: Does this property hold for the spherical geometry we discussed in class? Why or why not?
- What is the real projective plane? Make sure you know the definition, and how to work with it. What are the points? What are the lines? Sample question: How would you prove that any two points in the projective plane are contained in a unique line?
- How do constructions in projective geometry work? You only need to know the idea behind the ones we discussed; make sure you understand the relationship between projective geometry and constructions with straightedge. Sample question: 5.2.1 in the book.
- What is projection in \mathbb{R}^2 and \mathbb{R}^3 ? What kind of distortions can it produce? Sample question: Give an example of a projection from one line in \mathbb{R}^2 to another for which the distortion produces scaling by a fixed constant.
- What is the relationship between projections and fractional linear transformations? Sample question: Make sure you understand Figure 5.15 in your book.
- What are linear fractional functions? What is their relationship with the real projective line? What is the real projective line? What does it have to do with dividing by 0? Sample question: What does the transformation $f(x) = \frac{2x+3}{x+1}$ do when $x = 1$? $x = -1$? $x = \infty$?
- What is the cross-ratio? Why is it significant? Sample question: Show that the cross-ratio is preserved by $x \rightarrow 1/x$.

- What happens when you take the projective plane over different fields? For example, over a finite field? Sample question: How many points are there in \mathbb{Z}_2P^2 , where \mathbb{Z}_2 is the field of 2 elements that we described in class.
- Is there a unique geometry where any two lines intersect at a unique point? Sample question: Give an example answering this question.
- It would be nice to know a bit about projective spaces of other dimensions, in particular $\mathbb{R}P^3$. I wouldn't ask much about this, but...Sample question: What are points in $\mathbb{R}P^3$? What are lines? Do any two lines contain a unique point?