Mathematics 128A, Fall 2018
D. Cristofaro-Gardiner
Practice Midterm

1. Explain how to construct the following with straight-edge and compass.

   (a) The equilateral triangle on a given line segment.

   (b) The midpoint of a given line segment.

   (c) The bisector of a given angle.

2. The *triangle inequality* states that the sum of the lengths of any two different sides of a triangle is more than the length of the third side. To prove this in coordinates, we can assume that one vertex A is at $(0,0)$, another B is at $(x,0)$ for $x > 0$; here for simplicity we do the special case where the third C is at (x,y) for $y > 0$ – you should prove that

   $$|AB| + |BC| > |AC|. \tag{1}$$

   You do not need to prove any of what I said above – but do answer the following:

   (a) To prove (1), it is sufficient to show that $(|AB| + |BC|)^2 - |AC|^2 > 0$. Why?

   (b) What is $(|AB| + |BC|)^2 - |AC|^2$, as a function of x and y?

   (c) Why is $(|AB| + |BC|)^2 - |AC|^2 > 0$?

3. (a) Give the definition of an isometry.

   (b) Do isometries preserve angles? Why or why not? (In either case, give an explanation.)

   (c) Give an explicit formula for reflection about the line $y = x + 1$. (In other words, if the reflection is denoted by f, for every (x,y) tell me what $f(x,y)$ is.)

   (d) Explain how to write rotation by angle θ about the origin as a composition of reflections. (For this, you do not need to give explicit formulas; just tell me what the reflections are.)

4. Explain how to construct the following with straight-edge and compass.

   (a) A line parallel to a given line, through a given point.

(b) A line segment in the same direction as a given line segment, with the same length as that segment. (You can assume for this that you have a compass that stores length, although you should not assume that it stores direction; and, you should make sure you understand why we are justified via Euclid in assuming we have a compass that stores length.)

(c) A line segment of length $\sqrt{2}$, given a line segment of length 1. Please explain why your line segment has the claimed length.

5. (a) Let $\mathbf{a} = (2, 3)$ and $\mathbf{b} = (3, 4)$. What is $\mathbf{a} \cdot \mathbf{b}$?

(b) Let $\mathbf{v}, \mathbf{w}$ and $\mathbf{u}$ form a triangle. Where do the medians of the triangle with vertices $\mathbf{v}, \mathbf{w}$, and $\mathbf{u}$ intersect? You just have to state the answers; you do not need to prove anything.

(c) Recall that an *altitude* of a triangle is a line from a vertex to the opposite side that is perpendicular to that side. Show using vectors that the altitudes of any triangle intersect.

Extra credit: Show that for any triangle, the perpendicular bisectors intersect and moreover the intersection point of the medians, the intersection point of the altitudes, and the intersection point of the perpendicular bisectors lie on the same line.