Final project suggestions

Below are some suggestions for your (optional) final projects. You are **not** required to pick a project from this list — you can come up with your own project, and this list is only meant to help guide you.

(1) Give an expository account of **fundamental existence and uniqueness**: provide the **details** of the proof, and give examples where uniqueness and long-term existence fail.

(2) Explore the **numerical methods** we have been discussing in this class in greater depth. For example, one way to improve on the methods is by allowing for the step-size to be chosen adaptively. Explore this.

(3) Explain how to solve the **two-body problem**: this asks for the solution to a system consisting of two masses interacting under Newton's Law of Gravitation.

(4) Give an expository account of the **three-body problem**.

(5) Give more details about how to compute **matrix exponentials**. For example, explain how to find a Jordan basis, and explain what the matrix exponential of a matrix in Jordan Canonical Form is.

(6) Discuss the perspective on first order ODEs that comes from thinking of them as **flows of vector fields**. What is a flow? Under what conditions is the flow for some fixed time a *smooth map*? What does it mean for a flow to be area preserving?

(7) Elaborate on the **power series** methods that we discussed in class. For example, explain the proof that a single ODE can be solved locally as a power series under suitable conditions. And/or, explore extending the power series method to systems.

(8) **Write code** implementing some of the numerical models we have discussed in class — the friendlier your output for the user, the better. Include the code itself as part of your final project.

(9) Explain some **applications** of first order systems in the natural sciences. For example, you could explore more about the population models we discussed. Or, you could look for applications in other fields, like finance and biology.

(10) Report on recent **scientific literature** that uses ODEs in an essential way for modeling. For example, you could find a publication in, say, the field of biology that has an ODE model, and explain more about the paper.

(11) Report on **structural stability**, as described in Chapter 8* in our book. This is a long section, but you could for example try to explain some of the key ideas, and give some illustrative examples and/or proofs.

(12) Discuss **chaos**. For example, explore some of what is discussed in Chapter 8.7 of your book in more depth.

(13) Explore some aspects of **discrete** dynamical systems and **iteration**, as introduced for example in Chapter 5 of your book. There is a lot in this chapter, but there are many possible subsets of the material covered that could make interesting projects. For example, the material about cascades in section 5.1 is quite fascinating.

(14) Explore some aspects of **bifurcation theory**. For example, an explication of the illustrative Example 9.7.1 in your book would be great.

(15) Explore some of the theory of **ODEs on shapes** like the surface of a sphere or a donut. Part of your project could involve explaining the concepts of **vector fields on manifolds**.

(16) What are **Hamilton's ODEs** and why are they important? You could explain why they preserve area, and what this means; and/or, you could give some illustrative examples.

(17) What is a **configuration space**? Give some examples for some interesting systems, and write down some relevant ODEs.

(18) Explain the concept of **resonance** from the point of view of linear ODEs. Give some examples of its importance in applications, either as a desired phenomenon or as a phenomenon to be avoided.

(19) Explain applications of **Fourier series** to nonhomogeneous linear systems, from the point of view of the method of undetermined coefficients.

(20) Explain what the **Fourier** and/or **Laplace transforms** are, and explain how to use them to better understand nonhomogenous linear systems. (This is related to the previous project as well.)

(21) How common are **limit cycles**? Report on some conjectures and theorems about this.

(22) Explain some of Smale's classic article **Differentiable dynamical systems**. This is an advanced article, which is certainly at a graduate level, so this could be hard. But, you would only have to understand some aspects of it, and I could help you.

(23) Write about the connections between **dynamical systems** and **systems of ODEs**. What is a dynamical system? Give some examples.

(24) Go further into the theory of dynamical systems. For example, give an introduction to the **entropy theory** of dynamical systems. Or, discuss some features of **hyperbolic** dynamical systems.

(25) Write more about the solutions of some **interesting** ODEs. For example, further explore the ODE $x' = sin(tx)$ that we have discussed in class. Or, find some other interesting ODEs, especially nonlinear ones, that are amenable to analysis