

Dear students,

Some of you have asked: what skills do I need to have to succeed on the final? In terms of core skills, I would suggest that you feel you can answer the following questions:

Questions about systems:

- How do I compute the matrix exponential of a 2×2 or 3×3 matrix? (It is unlikely I will ask about the 3×3 case, for time reasons.)
- What does the matrix exponential have to do with linear systems?
- How do I use the method of undetermined coefficients to solve a non-homogeneous linear system?
- How do I use variation of parameters to solve a nonhomogeneous linear system?
- What are the zeros of a system, and why are they significant? How do I linearize a nonlinear system at its zeros?
- What are the most common types of zeros? What are node sources? Node sinks? Saddles? Spirals? How can I tell from the linearization what kind of type I have?
- How do I write the general solution of a 2×2 linear homogeneous system, assuming the matrix for the system is diagonalizable?
- How do I write particular solutions for 2×2 (and maybe 3×3 systems), satisfying a given initial condition?
- How do I plot solutions to a 2×2 system on a plane?
- What does the “uniqueness” part from the “existence and uniqueness” theorem say about the plots of solutions?
- What does it mean for a system to be linear, versus nonlinear?
- What does it mean for a system to be autonomous?
- How do I write a single higher order equation as a system?

- How do I write a nonautonomous system as an autonomous one?
- What is the relationship between autonomous systems and vector fields? How do I sketch vector fields? How do I sketch a solution of a system from a vector field?¹

Questions about a single equation:

- What is Euler's method, and midpoint Euler, and how do I use them?
- How do I use separation of variables, the power series method, the method of undetermined coefficients, and variation of parameters? What are exact equations and how do I solve them explicitly?
- What is an example of a model that uses a differential equation?
- What are fences? What do we learn from the existence of a fence? How can I show that a given curve is a fence?
- What are funnels? Anti-funnels? What do they look like?
- What are isoclines? How do I find them?
- What is the slope field, and how can I sketch it? How can I sketch a solution from the slope field?
- What is a vertical asymptote?
- What does fundamental existence and uniqueness say, at least generally, and when can uniqueness fail?

¹If you are confused about this, I encourage you to play with the tool on the website to explore this.