Mathematics 106; Winter 2018; D. Cristofaro-Gardiner
Practice Midterm

1. Consider the differential equation

$$x' = sin(tx).$$

   (a) Sketch 3 different isoclines: label the coordinates of one point on each, and label the value of $sin(tx)$ on each.

   (b) Sketch the slope field, by plotting at least 10 slopes.

   (c) Show that for all k, the curves

$$\alpha_k(t) = 2k\pi/t, \qquad \beta_k(t) = (2k - 1/2))\pi/t$$

   determine an anti-funnel.

2. Consider a population of size $V(t)$, subject to the differential equation

$$V'(t) = aV(t) - bV(t)^2,$$

where a and b are positive constants. (Recall that this is our model for "growth with competition".)

(a) Solve for $V(t)$ explicitly.

(b) What is the long term outlook for the population? In other words, what is the limit of V as t goes to infinity? Show your work.

(c) Now let's make our model a little more complicated: let's assume V is subject to

$$V'(t) = (2 + cos(t))x - (1/2)x^2 - 1.$$

We don't know how to solve this explicitly, so let's try one of our numerical schemes. Namely, do Euler's method twice, with $h = 1$. Start from the initial condition $t = 0, V(t) = 1000$. How many people are there at the end of this approximation?

3. Give examples of:

   (a) Upper and lower fences, defined for all t, for the differential equation

   $$x' = 2sin(xe^t).$$

   (b) A differential equation that is not separable.

   (c) A differential equation with a solution that develops a vertical asymptote.

4. (a) Solve:

$$(4x - t)x' = (x - 3t^2).$$

(b) Consider the differential equation

$$x' = x^2 - t.$$

Find a funnel, defined for sufficiently large t and asymptotic to $x = -\sqrt{t}$, whose upper fence and lower fence are **isoclines**.