

Dear students,

I was asked: how can we simplify expression using Euler's Identity?
Recall that Euler's Identity says that

$$e^{i\theta} = \cos(\theta) + i\sin(\theta).$$

There are two main ways that we want to use this. We learn from the identity that

$$e^{i\theta} + e^{-i\theta} = 2\cos(\theta),$$

because *cos* is an *even* function, while *sin* is *odd*. We also learn that

$$e^{i\theta} - e^{-i\theta} = 2i\sin(\theta),$$

for the same reason.

Example:

In class, I mentioned that we wanted to simplify

$$\frac{1}{4i}2(e^{(3+2i)t} - e^{(3-2i)t}).$$

This looks complicated, but it's actually totally real.

We simplify using the identities above as follows:

$$\begin{aligned} \frac{1}{4i}2(e^{(3+2i)t} - e^{(3-2i)t}) &= \frac{e^{3t}}{2i}(e^{2it} - e^{-2it}) \\ &= \frac{e^{3t}}{2i}(2i\sin(2t)) = e^{3t}\sin(2t) \end{aligned}$$