Math 106 - Systems of ordinary differential equations Dan Cristofaro-Gardiner
Winter 2018
Lectures: MWF, 12 - 1:05 PM, Engineering 2 192

Instructor Information:

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Office Hours M 3:00-4:50, W 2 - 3:10
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Course Description:

Equations involving derivatives are called differential equations, and such equations are of fundamental importance across mathematics and the sciences. Ordinary differential equations are differential equations where the derivatives involve functions of a single variable, which we will usually think of as time.

This course is about teaching you how to solve ordinary differential equations when you see them. We will start with single differential equations, and then move on to systems, and we will study both linear and nonlinear examples. Our emphasis will be on understanding the behavior of solutions as we vary initial conditions, and we will develop qualitative, analytical, and numerical tools for this purpose.

For more about the course, please read the "Goals" and "Pedagogy" sections of the syllabus.

Prerequisites:

To take this course, you should have definitely completed Math 21 or the equivalent. You should also have taken Math 100 or the equivalent, and ideally Math 24.

If you do not have all of these prerequisites, and would still like to take the course, I would encourage you to speak with me one on one.

Textbook:

The textbook for the course is Differential Equations: A Dynamical Systems Approach, by Hubbard and West. We will be using both Parts I and Parts II. The textbook is available in the bookstore.

Teaching assistant:

Our teaching assistant is Andres Perico (aperico@ucsc.edu). Andres will have office hours, and will be announcing a time and place for these office hours soon.

Email and Website:

There is a website for this course, at https://dancg.sites.ucsc.edu/teaching/math-106-systems-of-ordinary-differential-equations/. The homework for the course will be posted there, as will any essential announcements, and some useful online tools for analyzing differential equations. I might also post clarifying notes from time to time; for example, if many students ask me a similar question, I will post a response.

You are encouraged to email me, or the TA, with any questions that you might have. I will try to respond to all emails with 48 hours.

Discussion section:

Our teaching assistant will be running weekly discussion sections to complement the course. This should be a valuable part of your learning experience, and you **must enroll** in a section; the TA will sometimes give graded quizzes to test your understanding. (He will announce the dates of these quizzes at least a week in advance.) There are two sections: W 4:00-5:05 PM and W 8 - 9:05 AM. Sections begin on Tuesday, **January 16**.

Academic accommodations:

To receive academic accommodations for a physical or learning disability, please submit an Accommodation Authorization Letter from the Disability Resource Center (DRC) to me as soon as possible (ideally within the first two weeks of the quarter), and contact PBSci Testing at **testing.pbsci@ucsc.edu** for arrangements at least two weeks prior to any exam. If you do not currently have accommodations authorized, you will be referred to the Disability Resource Center (DRC). You may contact the DRC by phone at 459-2089, or by email at **drc@ucsc.edu**.

Grading rubric:

One midterm: 27.5%
One final: 37.5%
Quizzes in discussion section: 15%
Homework: 20%
Optional extra credit project: 10%

Late work:

Please note that except in exceptional circumstances with appropriate documentation, or in line with an academic accommodation, late homework will not be accepted and missed quizzes or midterms can not be retaken.

Homework:

Homework will be posted to the course webpage, with a due date, and will be due at the beginning of class. The first homework will be due **Wednesday, January 17**, and will be posted **Wednesday, January 10**. Your teaching assistant will grade and return your homework.

Goals for the course:

My hope for you is that by the end of the course you should:

- Understand what an ordinary differential equation is, and understand what it means for a function to solve it.

- Understand what a system of ordinary differential equations is, and understand what it means for a collection of functions to solve it.

- Understand what it means for a differential equation, or a system of differential equations, to have a solution, even if one can not explicitly solve the equation. What tools do we have for studying solutions that we do not have explicit formulas for?

- Understand how to visualize ordinary differential equations, or systems. How can we graphically represent ordinary differential equations in terms of slope fields? What does it mean to sketch a solution from a slope field?

- Understand how to use a computer to explore the solutions to a differential equation.

- Know some qualitative techniques for analyzing differential equations, or systems. What are fences? What are tunnels and anti-tunnels? What are vertical asymptotes?

- Know some techniques for explicitly solving certain differential equations, or systems. How can we recognize differential equations that can be solved explicitly, and what kind of techniques can we try to use? What are separable equations? What is the method of undetermined coefficients? What is the Taylor series method?

- Know some numerical techniques for producing approximate solutions to differential equations, or systems. What is Euler's method, and what is the Runge-Kutta method?

- Understand the meaning of the fundamental existence and uniqueness theorem.

- Understand some of the differences between a single differential equation, and a system of differential equations. Why do we call a single differential equation the one-dimensional case, and a system higher dimensional?

- Know some examples of interesting differential equations.

- Know what a conservation law is, and why it is important.

- Understand the linear theory, especially in the case of constant coefficients. What does superposition mean? Why are eigenvectors important? What kind of two-by-two linear systems can exist? How can nonhomogeneous equations be approached?

- Understand how to transform a higher order equation into a system of first order equations.

- Understand how to think of a system of first order equations in terms of the flow of a vector field, and in terms of the time evolution of a system.

- Know how to use linear techniques to better understand nonlinear systems. What is the linearization at a zero of a vector field?

- Understand some techniques for studying nonlinear systems. What are separatrices? Limit cycles? Basins and sources and sinks?

- Understand the basic idea behind structural stability,

- Understanding what can happen when differential equations have symmetries, or are volume preserving.

Pedagogy and advice:

For the most part, I will be lecturing during class time. However, I want to emphasize seven principles and tips that I think are very important for this course:

- Don't be afraid to ask questions in class! The more you engage with the material, the better your understanding will be. Often, the deepest understanding comes only after many mistakes.

- Do lots of examples and exercises.

- Try to build your visual understanding of the subject. I suggest playing with examples with the computer if you can.

- Try to have fun! I will try my best to make the course enjoyable and interesting, and I hope you will enjoy it too.

- When you are reading the textbook, focus on the **concepts**. For this class, the proofs of theorems are not very important. The statements of theorem are more important. The most important of all is to really understand in your bones what the theorems **mean**.

- Try not to fall behind. I think you will have an easier time if you keep on top of your work

- Come to office hours! It will be great to meet you.

Tentative lecture schedule (very much subject to change!):

- Qualitative methods (Textbook, Chapter 1): Weeks 1 and 2

- Analytic methods (Textbook, Chapter 2): Weeks 3 and 4

- Numerical methods and some aspects of fundamental existence and uniqueness (Textbook, Chapter 3, and DCG note): Weeks 4 and 5

- Introduction to higher dimensional systems (Textbook, Chapter 6): Week 6

- Higher dimensions: the linear theory (Textbook, Chapter 7): Weeks 7 and 8

- Higher dimensions: introduction to the nonlinear theory (Textbook, Chapter 8): Weeks 9 and 10

Lecture summaries:

I will periodically post very brief summaries of what was covered in lecture, with companion readings. See https://dancg.sites.ucsc.edu/teaching/math-232/math-232-lecture-summaries/ for an example of how this will look.

Key dates:

Midterm: Friday, February 9, in class
Final exam: 3/19, 4 - 7 PM