Math 232 - Morse theory Dan Cristofaro-Gardiner

Fall 2017

Instructor Information:

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Office Hours: Tuesday 1 - 3, or by appointment
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Course Description:

Morse theory seeks to relate the critical points of functions on a manifold to the topology of that manifold. This course will be an introduction to the subject. We will cover the fundamentals of Morse theory, and then move to Morse homology and its connections with various flavors of contact homologies.

For more about the course, please read the "Goals" section of this syllabus.

Prerequisites:

This course is meant as a "second course in topology" for graduate students. This means that you should definitely have some familiarity with the theory of smooth manifolds, and some basic familiarity with topology. It would be great if you were comfortable with ideas like singular and cellular homology too, but this is not required.

Especially in the later part of the course, we will need to assume some background material that you might not be too comfortable with; for example, some basics about characteristic classes. My presentations will be designed with the goal of being understandable even if you have not seen some of these ideas, but if you feel lost, you might have to do additional reading, or discuss with me during office hours.

If you are worried about prerequisites, I would encourage you to speak with me one on one.

My expectations for you:

My main expectation for you is that you will attend lectures; active participation is encouraged, but not required. There will be optional exercises stated throughout lecture. I strongly encourage you to work on these. You will also be expected to attend lectures; you might also enjoy writing a short expository paper about something you would like to explore in more depth, under my guidance. Alternatively, you might enjoy giving an expository talk in our symplectic seminar. Both of these activities are also optional, however.

Please also communicate with me if there is something that you would like to see covered in the course; or, alternately, if you are lost or confused about something. For more about this, please see the communications section.

Readings:

Here are some readings for the course. We will certainly not cover all of this. However, I will keep lecture summaries on the course website, that point out what sections of these books we covered.

I will also provide notes on the website about any major gaps between these references and what we cover in class.

Core texts:

Audin and Damian, *Morse theory and Floer homology*
Hutchings, *Lecture notes on Morse homology (with an eye towards Floer theory and pseudoholomorphic curves)* (available online)
Hutchings, *Lecture notes on embedded contact homology* (available online)
Hutchings and Nelson, *Cylindrical contact homology for dynamically convex contact forms in three dimensions* (available online)
Milnor, *Morse Theory* (available online)
Pardon, *Contact homology and virtual fundamental cycles* (available online)

Other very useful texts:

Bott, *Lectures on Morse Theory, Old and New* (available online)
Bott, *Morse theory indomitable* (available online)
Milnor, *Lectures on the h-cobordism theorem (*available online*)*

Goals for the course:

My hope for you is that by the end of the course you should:

- Understand the basics of classical Morse theory, as in the first part of Milnor's book. What is a Morse function? What does Morse's Lemma say, and what is its significance? Why do sub-level sets stay the same if we do not cross a critical value? What happens to homotopy type when we cross a critical value? What are the Morse inequalities, and why are they true?

- Understand the basics of Morse homology. What is the definition of Morse homology? What do the Morse inequalities look like from this perspective? What is a flow line? What is the moduli space of flow lines? What does the natural compactification of the moduli space of flow lines look like? How does the isomorphism with singular homology work? What are continuation maps, and what does this have to do with a priori invariance?

- Understand the basics of contact homology, and have some kind of idea of what goes into current research into the circle of ideas related to contact homology. What is a J-holomorphic curve? What goes wrong if we try to define a differential by counting cylinders, and we have no other assumptions? How might we try to fix this? What other perturbations do we have other than perturbations of the almost complex structure, and what kind of problems can come up when we try to achieve transversality this way?

- Understand a little bit about how contact homology can be applied. What is the Reeb vector field, and how can we use ideas from contact homology to study its closed orbits? What is a symplectic embedding, and how can we use contact homology to find symplectic embedding obstructions?

- Feel comfortable with all the proofs that we give in class.

- For theorems, propositions, or lemmas that we do not prove in class, feel like you understand the precise statement, and, ideally, some idea of why it is true.

- Feel comfortable with any "background" material that we used that you had not seen before. If you had not seen singular homology, do you understand the idea? If you had not seen characteristic classes before, do you understand the relative Chern class?

Communication:

We are lucky to have a small seminar course. I would like to take advantage of this, and of the graduate format of the course, to have some flexibility with what is covered. If there is something you would like to see, please let me know, and I will think about whether we can integrate it. I will also ask for your feedback periodically throughout the course.